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Empirical Didactical Design Using the Zone of Proximal Development to Address Learning Obstacles in Social Arithmetic

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Abstract

This study is motivated by the persistent learning obstacles students face in mastering social arithmetic, which hinder conceptual understanding and the ability to apply mathematical concepts to real-life contexts. The study aims to (1) identify students' learning obstacles in social arithmetic, (2) develop a didactical design based on the zone of proximal development that aligns with students' learning trajectories and obstacles, and (3) analyze the implementation of the design to produce an empirical didactical design that optimizes students' potential. The research was conducted at a public junior high school (SMP) in Indonesia and employed a didactical design research approach comprising three phases. The first phase, prospective analysis, involved identifying learning obstacles among 33 eighth-grade students and developing a Hypothetical Learning Trajectory (HLT) and an Analytical Design Plan (ADP). The second phase, meta-didactical analysis, consisted of implementing the didactical design with 29 seventh-grade students and analyzing their responses. The third phase, retrospective analysis, connected the findings from the implementation with the initial design hypothesis to develop an empirical didactical design. Data were collected through diagnostic tests, interviews, observations, and documentation. The results indicate that the developed didactical design effectively reduced students' learning obstacles, with a decrease of 0.51 in the obstacle score (moderate category). Some aspects require refinement, particularly in managing instructional duration. The design also enabled students to reach the actual development stage, fostered constructive social interactions, and facilitated the application of social arithmetic concepts in realistic contexts. These findings imply that didactical designs grounded in the zone of proximal development can serve as an effective strategy to enhance students' conceptual understanding and socio-mathematical skills, while providing a flexible framework that adapts to individual learning obstacles in Indonesian junior high schools.

Keywords: *Didactical Design Research, Empirical Didactical Design, Learning Obstacles, Social Arithmetic, Zone of Proximal Development*

A. Introduction

Mathematics is a fundamental discipline that underpins knowledge development and daily problem-solving. Empirical evidence from Indonesian junior high schools reveals a surprising phenomenon: many students struggle significantly with social arithmetic, perceiving it as abstract, difficult, and disconnected from their daily lives (Ana & Nusantara, 2021). Observations indicate that students often fail to connect arithmetic to real-world social contexts, resulting in low engagement and limited conceptual understanding. This aligns with Karnilah

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(2013), who reported that students often do not see mathematics as meaningful, which hinders the development of logical, critical, and analytical thinking essential for real-world problem-solving. Such difficulties demonstrate a persistent educational problem where mathematics learning does not adequately reflect students' social and cultural experiences.

Several studies have documented these challenges. Astutik (2015) identified three main types of errors in social arithmetic: conceptual, procedural, and technical errors, influenced by insufficient preparation, hurried responses, and lack of conceptual understanding. Similarly, Nurwindani et al. (2019) reported that conventional teaching methods often fail to address individual learning obstacles, leading to persistent difficulties in applying mathematics in real-life contexts. Internationally, studies by Swan (2006) and Boaler (2016) have shown that students' mathematical understanding improves when learning is contextualized and socially interactive, highlighting the importance of pedagogical strategies that consider both cognitive and social dimensions. However, most prior studies focus on identifying problems or errors rather than on providing empirically validated interventions that scaffold learning based on students' potential.

Despite these insights, the literature on didactical designs that systematically address learning obstacles in social arithmetic remains limited. Studies by Van de Walle et al. (2013) and Cai (2014) emphasize the need for instructional designs that adapt to students' developmental levels, yet few studies apply Vygotsky's Zone of Proximal Development (ZPD) to social arithmetic in secondary education contexts. Additionally, studies that incorporate iterative testing, student feedback, and retrospective analysis to refine instructional strategies are scarce, especially in Indonesian classrooms (Fitriani & Haryanto, 2020). This gap suggests a pressing need for studies that translate theoretical frameworks into practical, evidence-based designs.

To overcome these limitations, this study proposes an empirical didactical design grounded in ZPD, which aligns instructional strategies with students' developmental potential and observed learning obstacles (Vygotsky, 1978; Hammond & Gibbons, 2005). The design is developed iteratively through three phases: prospective analysis to identify learning obstacles, meta-didactical implementation to evaluate instructional effectiveness, and retrospective refinement to produce an empirically validated framework. This approach ensures both cognitive scaffolding and socially relevant learning experiences.

The focus of this study is threefold: (1) to identify specific learning obstacles in social arithmetic among Indonesian junior high students, (2) to develop a ZPD-based didactical design tailored to these obstacles and students' learning trajectories, and (3) to evaluate the effectiveness of the design in improving conceptual understanding, problem-solving skills, and social interaction in mathematics learning. By addressing both cognitive and social aspects, this study advances current pedagogical practices and provides a framework for more effective mathematics instruction.

This study is significant because it not only addresses persistent learning difficulties but also provides a replicable, empirically tested framework. Implementing a ZPD-based didactical design can reduce cognitive and social barriers, foster collaborative learning, and enhance students' ability to apply mathematical concepts in realistic contexts. Furthermore, this study contributes to the literature by shifting the focus from merely identifying errors to systematically reducing learning obstacles through tailored, evidence-based instructional design (Astutik, 2015; Nurwindani et al., 2019; Boaler, 2016).

B. Methods

This study employed a qualitative descriptive approach with a Didactical Design Research (DDR) design. A qualitative approach was selected to provide an in-depth description of students' learning obstacles in social arithmetic and to explore the implementation of the developed didactical design (Pramudyani, 2018; Sugiyono, 2015). The DDR design was adopted because it addresses instructional problems teachers encounter while simultaneously producing a learning design tailored to students' needs (Fauzi & Suryadi, 2020). The study was conducted during the even semester of the 2021/2022 academic year at SMP Negeri 1 Kertasemaya, Indonesia, specifically in Indramayu, West Java. The initial identification subjects consisted of 33 eighth-grade students (Class VIII A) who had previously studied social arithmetic, while the implementation subjects comprised 29 seventh-grade students (Class VII F). The object of this study was to identify students' learning obstacles in social arithmetic.

The research procedures followed the three main stages of DDR as outlined by Suryadi (2013). The first stage, prospective analysis, included: re-personalization of teaching materials; re-contextualization through teacher interviews and an initial learning obstacle (LO) diagnostic test; development of the diagnostic test instrument; identification of learning obstacles; student interviews; and the development of a hypothetical didactical design consisting of a Hypothetical Learning Trajectory (HLT) and Didactical-Pedagogical Anticipations (DPA). The second stage, meta-didactical analysis, involved implementing the didactical design with 29 seventh-grade students (Class VII F) from the same school in Indonesia by forming small learning groups integrated with the Zone of Proximal Development (ZPD) approach, followed by direct observation and analysis of student responses. The third stage, retrospective analysis, compared the initial and final LO test results to evaluate the design's effectiveness using the normalized gain formula adapted from Hake. This stage also produced an empirically validated revised didactical design.

Four data collection techniques were employed in this study. First, a diagnostic test comprising 7 essay items was administered to 33 eighth-grade students in Indonesia to identify their learning obstacles in social arithmetic (Hadi, 2015). Second, semi-structured and informal interviews were conducted with both teachers and students to gain deeper insights into the learning obstacles and students' conceptual understanding. Third, direct observation was carried out during the implementation of the didactical design to systematically record classroom situations, student responses, and teacher anticipatory actions. Fourth, documentation was collected, including photographs, student worksheets (LKPD), and other relevant documents as complementary and empirical evidence. All instruments were validated using the Content Validity Ratio (CVR) by five expert panelists (three mathematics education lecturers, one mathematics teacher, and one Indonesian linguist) and were declared valid and appropriate for use.

Data analysis was conducted in three stages aligned with the DDR framework. In the prospective analysis, data from the diagnostic test and interviews were analyzed to identify the types and sources of students' learning obstacles, which then served as the foundation for developing the hypothetical didactical design and learning trajectory. In the meta-didactical analysis, observational data and student responses during the implementation were analyzed by comparing the predicted student responses with actual classroom events. In the retrospective analysis, the effectiveness of the didactical design was measured by comparing the percentage of initial LO and final LO. The reduction level was categorized as high ($e \geq 0.7$), moderate ($0.3 \leq e \leq 0.7$), low ($0 \leq e \leq 0.3$), or ineffective ($e \leq 0$). The results of this analysis were subsequently used to revise the didactical design into an empirically improved design.

C. Results and

Identification of Specific Learning Obstacles in Social Arithmetic Among Indonesian Junior High Students

Textbook Analysis and Learning Trajectory

The prospective analysis began with an examination of four mathematics textbooks commonly used at SMP Negeri 1 Kertasemaya, Indonesia. The analysis revealed that students' learning trajectory in social arithmetic was constructed progressively across four major conceptual domains: profit and loss, discount and gross–net–tare calculations, taxation, and simple interest. However, significant variations in instructional sequencing and conceptual emphasis were observed across curricula. Textbooks based on the 2013 Curriculum tended to emphasize contextual exercises and problem-solving activities, whereas KTSP-based textbooks provided more detailed explanations of prerequisite mathematical concepts and procedural examples. Interviews with mathematics teachers indicated that the integration of multiple learning resources was intentionally conducted to complement conceptual explanations, enrich practice exercises, and facilitate students' understanding of social arithmetic concepts. Based on this analysis, the researcher formulated a learning trajectory consisting of four sequential stages: (1) unit value, profit, loss, and percentage of profit and loss; (2) discount, gross, net, and tare; (3) taxation; and (4) simple interest.

Table 1. Comparative Analysis of Social Arithmetic Instruction in the 2013 Curriculum and KTSP Mathematics Textbooks

Aspect	2013 Curriculum Textbooks	KTSP Textbooks
Instructional Emphasis	Focused on contextual learning and problem-solving activities.	Focused on detailed conceptual explanations and procedural examples.
Learning Activities	Encouraged students to solve real-life social arithmetic problems through discussion and application tasks.	Emphasized step-by-step calculation procedures and mastery of prerequisite concepts.
Presentation of Concepts	Concepts were introduced through contextual situations and student-centered exploration.	Concepts were explained directly with detailed formulas and worked examples.
Practice Exercises	Provided more varied contextual and higher-order thinking problems.	Provided structured and procedural practice questions.
Role in Learning Design	Supported the development of students' reasoning and contextual understanding.	Supported students' procedural fluency and foundational mathematical skills.
Implication for Prospective Analysis	Helped formulate contextual and discussion-based learning activities.	Helped identify prerequisite materials and procedural scaffolding needs.

The prospective analysis of four mathematics textbooks from the 2013 Curriculum and KTSP revealed complementary strengths that collectively inform an optimal learning trajectory for social arithmetic. Textbooks based on the 2013 Curriculum emphasized contextual exercises and problem-solving activities, introducing concepts through real-life situations and student-centered exploration, which supports the development of reasoning and transferable knowledge as advocated by constructivist theory (Piaget, 1970; Vygotsky, 1978). In contrast, KTSP-based textbooks provided detailed explanations of prerequisite mathematical concepts and procedural examples, emphasizing step-by-step calculations and mastery of foundational skills, aligning with cognitive information processing theories that prioritize procedural fluency before advancing to complex applications (Gagné, 1985; Anderson, 1995). Teacher interviews confirmed that the intentional integration of multiple learning resources was conducted to complement conceptual explanations, enrich practice exercises, and facilitate students' understanding of social arithmetic concepts. This finding supports Remillard's (2005) notion of "curricular flexibility," where effective teachers act as curricular interpreters rather than passive implementers, combining the contextual richness of the 2013 Curriculum with the procedural clarity of KTSP textbooks to address both dimensions of mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001).

Based on this comparative analysis, the researcher formulated a four-stage learning trajectory that logically progresses from basic to advanced concepts: (1) unit value, profit, loss, and percentage of profit and loss; (2) discount, gross, net, and tare; (3) taxation; and (4) simple interest. This hierarchical progression reflects Gagné's (1968) learning hierarchies, where prerequisite skills must be mastered before higher-order concepts can be successfully acquired, and aligns with Piaget's theory of cognitive development as students in early adolescence transition from concrete operational to formal operational stages, enabling them to handle abstract concepts such as percentages and proportional reasoning with adequate scaffolding (Inhelder & Piaget, 1958). The 2013 Curriculum textbooks' emphasis on contextual learning supports students' ability to transfer mathematical knowledge to real-world situations (Boaler, 2016; Swan, 2006), while KTSP textbooks' focus on procedural mastery ensures students possess the foundational skills necessary for successful transfer (Hiebert & Grouws, 2007). Therefore, the integration of both approaches, as practiced by teachers at SMP Negeri 1 Kertasemaya and formalized in the four-stage trajectory, represents a balanced instructional strategy that aligns with the "balanced mathematics instruction" framework (National Mathematics Advisory Panel, 2008) and provides a coherent framework for developing didactical interventions within the Zone of Proximal Development (Vygotsky, 1978), where authentic real-life contexts serve as scaffolding tools to facilitate students' movement from potential to actual development levels.

Diagnostic Test Findings and Epistemological Obstacles

The recontextualization stage, conducted through a diagnostic learning obstacle test involving 33 eighth-grade students, revealed that epistemological obstacles constituted the primary source of students' difficulties in learning social arithmetic. These obstacles emerged because students' mathematical understanding was limited to familiar contexts, making it difficult for them to transfer concepts to new situations or contextual word problems (Brousseau, 2002). The highest percentage of learning obstacles was identified in gross, net, and tare concepts (87.88%), followed by simple interest (75.75%) and discount calculations (72.73%). This finding indicates that students' understanding remained predominantly procedural rather than conceptual, particularly in percentage-based calculations and multi-step contextual problems. Similar findings were reported by Rosita and Maharani (2020), who argued that epistemological obstacles arise when students' knowledge is restricted to limited contexts and cannot be generalized to broader problem situations.

Teacher interviews further confirmed that students tended to be passive during classroom instruction, reluctant to ask questions, and frequently struggled with percentage operations and contextual mathematical tasks. Most procedural errors were strongly associated with insufficient mastery of prerequisite mathematical skills, including multiplication, division, fractions, and percentage operations. As a result, many students failed to distinguish between selling price and purchase price, misapplied profit percentage formulas, and performed arbitrary calculations in discount and tax problems. This pattern supports the meaningful learning theory proposed by David Ausubel (1968), which emphasizes that new knowledge can only be meaningfully constructed when learners possess adequate prior conceptual foundations.

Table 2. Learning Obstacles and Didactical Design Development in Social Arithmetic Learning

Stage	Focus	Main	Didactical Design
Repersonalization	Analysis of textbooks and learning trajectories	The 2013 Curriculum emphasized contextual problems, while KTSP emphasized conceptual and procedural explanations. The learning trajectory consisted of profit/loss, discount, gross-net-to-tara, taxation, and simple interest.	A four-stage learning sequence was developed based on curriculum and textbook analysis.
Recontextualization	Identification of learning obstacles	Students experienced difficulties in percentage operations, contextual word problems, and prerequisite mathematical skills. High learning obstacles were found across all social arithmetic concepts.	Diagnostic test results and interviews were used to design learning interventions and scaffolding strategies.
Meeting 1	Profit, loss, and percentage profit/loss	Students struggled to determine the selling price, purchase price, and percentage calculations.	Learning used contextual examples, prerequisite reviews, group discussions, and peer tutoring within the ZPD framework.
Meeting 2	Discount, bruto, netto, and tara	Students experienced misconceptions about discounts and difficulties with percentage multiplication.	Learning used marketplace and packaging examples combined with collaborative problem-solving activities.
Meeting 3	Taxation	Students had difficulties understanding tax calculations and multi-step percentage operations.	Supermarket receipts were used as contextual media, supported by group discussions and scaffolding.
Meeting 4	Simple interest	Students struggled with simple interest calculations and contextual financial problems.	Online loan examples, peer tutoring, and collaborative problem-solving activities were implemented.

The didactical design was systematically developed across four instructional meetings, each targeting specific learning obstacles while integrating ZPD-based scaffolding strategies. As shown in Table 2, the repersonalization stage established a four-stage learning trajectory (profit/loss, discount, and bruto-netto-tara, taxation, and simple interest) derived from the comparative analysis of the 2013 Curriculum and KTSP textbooks. The recontextualization stage then used diagnostic test results and teacher interviews to design targeted interventions, including contextual examples, prerequisite reviews, collaborative group discussions, and peer tutoring within the ZPD framework (Vygotsky, 1978). Each meeting addressed specific obstacles: Meeting 1 focused on students' difficulties in determining selling price, purchase price, and percentage calculations through contextual examples and prerequisite reviews; Meeting 2 targeted misconceptions about discounts and percentage multiplication using marketplace and packaging examples combined with collaborative problem-solving; Meeting 3 addressed tax calculation difficulties using supermarket receipts as contextual media supported by group discussions and scaffolding; and Meeting 4 focused on simple interest and contextual financial problems through online loan examples and peer tutoring. This progressive, obstacle-driven design reflects the principles of didactical engineering (Artigue, 2015), in which instructional decisions are empirically grounded in students' actual learning difficulties rather than on theoretical assumptions alone. By systematically aligning each instructional component with the specific epistemological obstacles identified in the diagnostic phase, the design ensures that scaffolding is both targeted and developmentally appropriate, facilitating students' transition from potential to actual development levels within the ZPD framework (Hammond & Gibbons, 2005).

Patterns of Misconceptions and Procedural Errors

The diagnostic analysis also demonstrated diverse patterns of students' misconceptions and procedural errors across each indicator of social arithmetic. In the concepts of profit and loss, students frequently failed to distinguish between selling price and purchase price, leading to incorrect determinations of gain or loss. In percentage and taxation problems, several students identified relevant formulas but failed to correctly apply percentage multiplication procedures. Meanwhile, in discount and gross–net–tare problems, students often performed arbitrary arithmetic operations without understanding the conceptual relationships embedded in the problems. Similarly, in simple interest problems, students had difficulty interpreting the relationship between interest rates and the duration of savings. Interview data further revealed that some students relied on intuitive calculations rather than conceptual reasoning. These findings suggest that students' mathematical understanding remained predominantly procedural rather than conceptual, limiting their ability to solve contextual social arithmetic problems accurately. A complete summary of learning obstacle percentages is presented in the table.

Table 3. Didactical Design Development Based on Students' Misconceptions and Procedural Errors in Social Arithmetic Learning

Meeting	Topic	Patterns of Misconceptions and Procedural Errors	Didactical Design and HLT	Didactical and Pedagogical Anticipation (DPA)
1	Unit Value, Profit, Loss, Percentage of Profit and Loss	Students had difficulty distinguishing between the selling price and the purchase price, leading to incorrect profit or loss determinations. Students also made procedural errors in percentage operations.	Learning activities used contextual snack-related examples, reinforcement of prerequisite concepts, digital worksheets, group discussions, and ZPD-based peer tutoring.	The teacher provided scaffolding, re-explained profit–loss and percentage concepts, and guided group discussions and student presentations.

Meeting	Topic	Patterns of Misconceptions and Procedural Errors	Didactical Design and HLT	Didactical and Pedagogical Anticipation (DPA)
2	Discount, Gross, Net, and Tare	Students performed arithmetic operations randomly without understanding the conceptual relationships among discount, gross, net, and tare. Procedural errors also occurred in percentage operations and word problems.	Learning activities used marketplace and product packaging examples, contextual problem-solving activities, digital worksheets, and collaborative group discussions.	The teacher provided concrete examples, guided students to understand conceptual relationships, and reinforced concepts through group discussions.
3	Taxation	Students were able to identify tax formulas but made errors in percentage multiplication and in multi-step problem-solving.	Learning activities used supermarket receipts, contextual taxation examples, digital worksheets, and group problem-solving activities.	The teacher provided step-by-step explanations of percentage operations, scaffolding for word problems, and concept reinforcement through reflection activities.
4	Simple Interest	Students had difficulty understanding the relationships among the interest amount, interest rate, and loan or savings duration. Some students relied on intuitive calculations rather than conceptual reasoning.	Learning activities used online loan examples, digital worksheets, ZPD-based group discussions, and contextual problem-solving activities.	The teacher provided simple real-life examples, guided students in interpreting relationships among simple-interest variables, and scaffolded during group discussions.

The diagnostic analysis indicated that students' misconceptions and procedural errors in social arithmetic were largely caused by a procedural rather than a conceptual understanding of mathematics. In profit-and-loss problems, students often fail to distinguish between the selling price and the purchase price, leading to incorrect profit or loss determinations. This finding supports Astutik (2015), who argued that conceptual misunderstandings in social arithmetic often arise when students memorize formulas without understanding the underlying economic relationships. Likewise, in percentage and taxation problems, students were generally able to identify formulas but failed to apply percentage multiplication procedures correctly, reflecting only surface-level understanding rather than deep conceptual comprehension, as explained by Thomas P. Carpenter, Hiebert, and Pierre Lefevre (1986). In discount, gross–net, and tare problems, many students applied arithmetic operations randomly without understanding the relationships among the concepts. For instance, students often subtract percentages directly from prices without recognizing that discounts reduce prices multiplicatively. This tendency reflects what Richard Skemp (1976) described as instrumental understanding rather than relational understanding. Furthermore, in simple interest problems, students struggled to interpret the relationship between the interest rate and the time period, often applying annual interest rates directly to monthly periods without proportional reasoning. Interview findings also showed that some students relied on intuitive calculations instead of conceptual reasoning, supporting the findings of Nuraeni, Ardiansyah, and Zanthy (2020) that students' understanding of social arithmetic remains predominantly procedural. Collectively, these findings demonstrate the presence of epistemological obstacles: students are unable to transfer mathematical knowledge into unfamiliar contexts because they lack conceptual frameworks that connect procedures to real-life meanings, as proposed by Guy Brousseau (2002).

Based on these patterns of misconceptions and procedural errors, the didactical design was developed across four instructional meetings, each integrating targeted Hypothetical Learning Trajectories (HLT) and Didactical-Pedagogical Anticipations (DPA), as summarized in Table 3. In Meeting 1, contextual snack-related examples, prerequisite reinforcement, and ZPD-based peer tutoring were used to address students' difficulty distinguishing between selling and purchase prices. In Meeting 2, marketplace and product packaging examples were employed to clarify the conceptual relationships among discount, gross, net, and tare through collaborative problem-solving activities. Meeting 3 focused on taxation by using supermarket receipts and step-by-step scaffolding to improve students' understanding of percentage multiplication and multi-step calculations. Meanwhile, Meeting 4 addressed misconceptions about simple interest using online loan examples that helped students interpret the relationships among the interest amount, rate, and duration. Across all meetings, DPA strategies such as scaffolding, concept re-explanation, guided discussions, and student presentations were implemented to anticipate and respond to students' errors during instruction. This design reflects the principles of didactical engineering proposed by Michèle Artigue (2015), emphasizing instructional interventions grounded in students' actual learning obstacles. Through these strategies, the learning process was designed to support students' transition from instrumental to relational understanding (Skemp, 1976) and from potential to actual development within the Zone of Proximal Development framework developed by Lev Vygotsky (1978).

Development of a ZPD-Based Didactical Design Tailored to Learning Obstacles and Students' Learning Trajectories

Design Framework and Theoretical Grounding

Based on the identified learning obstacles, the researcher developed a didactical design and Hypothetical Learning Trajectory (HLT) consisting of four instructional meetings, each addressing one of the four sequential stages of the learning trajectory. Each instructional design incorporated predicted student responses and Didactical-Pedagogical Anticipation (DPA) strategies to address potential misconceptions during the learning process. The instructional activities were designed through contextual learning approaches, digital student worksheets, collaborative discussions, and problem-solving tasks grounded in students' daily experiences, such as online shopping discounts, supermarket taxation, and loan interest calculations. These instructional strategies were theoretically grounded in three complementary frameworks: Ausubel's meaningful learning theory (emphasizing the connection between new and prior knowledge), constructivist learning theory (enabling students to actively build their own understanding), and Vygotsky's Zone of Proximal Development (ZPD) (facilitating learning through social interaction and scaffolding from more capable peers).

Table 4. Didactical Design Framework and Implementation in Social Arithmetic Learning

Meeting	Topic	Main Learning Obstacles	Learning Design and Theoretical Grounding	Results
1	Profit, loss, and percentages	Students struggled to distinguish between selling and purchase prices and experienced difficulties with percentage operations.	Contextual snack examples, prerequisite reviews, collaborative discussions, and peer tutoring based on meaningful learning, constructivism, and ZPD theory.	Most students were able to solve profit-and-loss problems more independently.
2	Discount, gross, net, and tare	Students confused discount, gross, net, and tare concepts and made procedural calculation errors.	Marketplace and product-packaging contexts, collaborative problem-solving, and digital worksheets grounded in contextual and constructivist learning.	Students showed better conceptual understanding and participation during discussions.
3	Taxation	Students experienced difficulties in percentage multiplication and multi-step tax calculations.	Supermarket receipt examples, group discussions, scaffolding, and contextual problem-solving activities based on meaningful learning and ZPD.	Most students improved in solving taxation problems correctly.
4	Simple interest	Students struggled to understand relationships among interest, rate, and time period.	Online loan examples, collaborative discussions, and contextual tasks supported by scaffolding and peer interaction.	Students demonstrated improved understanding of simple interest calculations.

The didactical design developed in this study integrated three complementary theoretical frameworks—Ausubel's meaningful learning theory, constructivist learning theory, and Vygotsky's Zone of Proximal Development (ZPD)—to systematically address the identified learning obstacles across four sequential instructional meetings, as presented in Table 4. The integration of these frameworks is particularly significant because each addresses a distinct dimension of the learning process: Ausubel's (1968) theory ensures that new knowledge (e.g., discount calculations, tax computations) is meaningfully connected to students' prior knowledge (e.g., basic percentage operations) through contextual examples such as online shopping discounts, supermarket receipts, and loan interest calculations; constructivist theory (Piaget, 1970) positions students as active builders of their own understanding through collaborative discussions, digital worksheets, and problem-solving tasks grounded in their daily experiences; and Vygotsky's (1978) ZPD framework facilitates learning through social interaction and scaffolding from more capable peers, operationalized through peer tutoring and teacher-guided group discussions. The results summarized in Table 4 demonstrate the effectiveness of this integrated approach: in Meeting 1 (profit, loss, and percentages), contextual snack examples and peer tutoring enabled most students to solve profit and loss problems more independently; in Meeting 2 (discount, gross, net, and tare), marketplace and product-packaging contexts combined with collaborative problem-solving improved students' conceptual understanding and participation; in Meeting 3 (taxation), supermarket receipt examples and scaffolding within the ZPD framework helped most students solve taxation problems correctly; and in Meeting 4 (simple interest), online loan examples and peer interaction led to improved understanding of interest calculations. This progressive improvement across meetings supports the assertion by Hammond and Gibbons (2005) that effective scaffolding must be both calibrated to students' current developmental levels and gradually withdrawn as competence increases. Moreover, the incorporation of Didactical-Pedagogical Anticipation (DPA) strategies—which involve predicting student responses and preparing proactive interventions for potential misconceptions—aligns with Artigue's (2015) concept of didactical engineering, in which instructional design is empirically grounded in students' actual error patterns. The successful reduction of learning obstacles across all four topics confirms that the synergistic application of meaningful learning, constructivist, and ZPD frameworks creates a robust theoretical foundation for didactical design in social arithmetic, addressing both cognitive (conceptual understanding and procedural fluency) and social (collaboration and peer scaffolding) dimensions of mathematics learning.

Instructional Components and ZPD Integration

The didactical design operationalized ZPD principles by establishing heterogeneous small groups, each headed by a student who had reached the actual developmental level. Prior to each discussion session, these group leaders received intensive teacher scaffolding, which equipped them to effectively assist classmates still functioning at the potential development level. Digital worksheets developed on the Livelearning platform provided students with instructional videos, content summaries, and practice exercises accessible before each meeting. A three-stage structure was followed in every instructional session: an opening phase that employed authentic real-world artifacts to stimulate prior knowledge and curiosity; a core phase characterized by collaborative group discussions and problem-solving tasks; and a closing phase involving reflection, synthesis of conclusions, and completion of independent homework assignments. Additionally, Didactical-Pedagogical Anticipation (DPA) strategies—including dialogic scaffolding, concept review, and motivational reinforcement—were embedded in the design to proactively address the specific learning obstacles identified during the diagnostic stage. This systematic integration of ZPD principles is consistent with Vygotsky's (1978) proposition that learning occurs first at the social level through interaction with more knowledgeable others, followed by independent internalization. The purposeful assignment of group leaders who had already reached the actual development level operationalizes the function of the "more knowledgeable other" (MKO) within the ZPD framework, and the intensive teacher scaffolding delivered to these leaders before discussions ensures that they possess sufficient conceptual mastery to guide their peers effectively (Wood, Bruner, & Ross, 1976). Moreover, the three-phase instructional structure (opening, core, closing) mirrors the gradual release of responsibility model (Pearson & Gallagher, 1983), in which the teacher first activates prior knowledge and models reasoning, then facilitates collaborative practice through group discussions, and ultimately transfers responsibility to students through independent homework. The incorporation of digital worksheets on the Liveworksheets platform also meets the demand for flexible, self-regulated learning, enabling students to revisit prerequisite concepts and instructional materials before face-to-face sessions, thereby optimizing the effectiveness of in-class collaborative activities (Means et al., 2014).

From the perspective of sociocultural theory, which posits that cognitive development is mediated by social interaction and cultural tools, the effectiveness of this ZPD-integrated instructional design becomes evident (Vygotsky, 1978; Wertsch, 1991). The heterogeneous grouping strategy employed in this study addresses a fundamental limitation of conventional ability grouping, which often perpetuates existing achievement gaps by isolating struggling learners from peers who serve as role models (Slavin, 1996). In contrast, positioning students who have achieved the actual development level as group leaders creates natural opportunities for modeling, explanation, and guided practice within the proximal development zone of their peers. Teacher interviews and classroom observations confirmed that this peer-assisted learning strategy diminished students' hesitation to ask questions and enhanced their willingness to present group findings, as learners felt more at ease receiving explanations delivered in familiar language and communication styles by their peers. This result aligns with Tharp and Gallimore's (1988) four-stage model of ZPD progression: (1) assistance from more capable others (teacher scaffolding to group leaders), (2) self-assistance (group leaders internalizing concepts), (3) internalization and automatization (group leaders guiding peers without external prompts), and (4) de-automatization (students solving problems independently without assistance). The observation that by the third and fourth meetings, fewer groups showed over-dependence on specific individuals, and students demonstrated greater autonomy in completing homework tasks, indicates successful movement through these stages. Furthermore, the DPA strategies incorporated into the design—particularly dialogic scaffolding and motivational reinforcement—address the fluid nature of the ZPD, where the degree and form of assistance must be continuously adjusted in response to students' real-time reactions (Hammond & Gibbons, 2005). This responsive approach ensures that scaffolding is neither excessive (which would cultivate dependency) nor insufficient (which would cause frustration and disengagement), but rather calibrated to maintain students within their optimal learning zone (Vygotsky, 1978). The documented reduction of learning obstacles across all four instructional meetings, as presented in Table 2, provides empirical evidence that the combination of heterogeneous grouping, peer-assisted learning, digital preparatory materials, and responsive DPA strategies constitutes an effective operationalization of ZPD principles for mathematics instruction in Indonesian secondary classrooms.

Alignment with Learning Trajectory and Student Needs

The developed didactical design was systematically aligned with the four-stage learning trajectory identified through textbook analysis and diagnostic assessment. Each instructional meeting focused on a specific topic in social arithmetic and directly addressed the dominant misconceptions and procedural errors students encountered. The first meeting focused on buying and selling transactions, including unit value, profit, loss, and percentage calculations, because many students were unable to distinguish between the selling price and purchase price and frequently misapplied percentage operations. The second meeting focused on discount, gross, net, and tare concepts, which represented the highest percentage of learning obstacles identified during the diagnostic phase. The third meeting addressed taxation concepts through contextual learning activities using supermarket receipts, while the fourth meeting focused on simple interest by utilizing online loan advertisements to connect abstract concepts with students' daily experiences. Across all meetings, the design consistently implemented scaffolding strategies, collaborative learning, and contextual problem-solving activities to support students' conceptual understanding.

Table 5. Alignment of Didactical Design, Learning Trajectory, and Learning Obstacles

Meeting	Learning Topic	Main Learning Obstacles	Contextual Learning Media	Instructional Strategy	Learning Outcome
1	Unit value, profit, loss, and percentages	Difficulty distinguishing selling and purchase prices; errors in percentage operations	Snack products and buying-selling activities	Prerequisite review, peer tutoring, group discussion, scaffolding	Students improved their understanding of profit-loss concepts and contextual calculations
2	Discounts, gross, net, and tare	Random arithmetic operations and misunderstanding of concept relationships	Marketplace discounts and product packaging	Collaborative problem solving and guided discussion	Students better understood discount calculations and weight concepts
3	Taxation	Errors in percentage multiplication and multistep procedures	Supermarket receipts	Step-by-step explanation, reflection, and contextual exercises	Students improved their accuracy in tax calculations
4	Simple interest	Difficulty relating interest, rate, and time duration	Online loan advertisements	Guided inquiry, contextual discussion, and gradual reduction of support	Students became more independent in solving simple interest problems

The findings demonstrate that alignment among the learning trajectory, diagnostic findings, and instructional design reduced students' learning obstacles across all meetings. The use of authentic real-life contexts helped students connect mathematical concepts with everyday experiences, making abstract concepts more meaningful and easier to understand. This finding supports the view that contextual learning can reduce epistemological obstacles by linking mathematical procedures with their real-world applications (Brousseau, 2002). In addition, the implementation of collaborative discussions and peer-assisted learning enabled students to learn through social interaction, while scaffolding strategies supported students who still experienced conceptual difficulties. These instructional practices are consistent with the concept of scaffolding, where teacher assistance and peer tutoring help students gradually achieve independent understanding (Wood et al., 1976; Hammond & Gibbons, 2005). The gradual reduction of teacher assistance across meetings also encouraged students to develop greater independence and confidence in solving contextual social arithmetic problems. Furthermore, the sequential arrangement of learning materials from basic buying-selling concepts to more complex taxation and simple interest topics reflects the hierarchical nature of learning, in which mastery of prerequisite concepts is necessary before students can understand more advanced concepts (Gagné, 1968). The integration of contextual examples from students' daily experiences also strengthened students' perception that mathematics is meaningful and applicable in real life, supporting the perspective that contextual mathematics learning increases students' engagement and understanding (Karnilah, 2013). Overall, these findings indicate that the developed didactical design was not only theoretically grounded but also practically responsive to students' actual learning needs in mathematics learning.

Evaluation of the Effectiveness of the ZPD-Based Didactical Design in Improving Conceptual Understanding, Problem-Solving Skills, and Social Interaction

Improvement in Conceptual Understanding and Reduction of Learning Obstacles

To evaluate the effectiveness of the implemented didactical design, a post-implementation learning obstacle (LO) test was administered to the same group of 24 students. The results were then compared with the initial diagnostic test to examine changes in students' learning obstacles across various social arithmetic concepts. The findings demonstrated a substantial reduction in the percentage of students experiencing difficulties in nearly all indicators, indicating that the didactical interventions and ZPD-based scaffolding strategies successfully improved students' conceptual understanding.

Table 6. Comparison of Students' Learning Obstacles

Social Arithmetic Concept	Initial LO (%)	Final LO (%)	Reduction (%)	Interpretation
Profit and Loss	66.67	20.84	45.83	Significant reduction
Percentage of Profit and Loss	66.67	16.67	50.00	Significant reduction
Tax	60.60	29.17	31.43	Moderate reduction
Discount	72.73	41.67	31.06	Moderate reduction
Simple Interest	75.75	29.17	46.58	Significant reduction
Overall N-Gain	—	—	**0.58**	**Moderate effectiveness**

The substantial reduction in learning obstacles across all social arithmetic concepts, with an overall normalized gain of 0.58 (moderate), provides empirical evidence that the ZPD-based didactic design effectively improved students' conceptual understanding. The most pronounced improvements were observed in percentage profit/loss (50.00% reduction) and simple interest (46.58% reduction), suggesting that the design's emphasis on contextual examples and peer-assisted learning was particularly beneficial for topics requiring proportional reasoning and multi-step calculations (Hake, 1999). The moderate gain categorization, while not high, is consistent with findings from similar didactical intervention studies in mathematics education, where conceptual change typically requires sustained instructional exposure rather than short-term interventions (Hattie, 2009). Notably, the persistent learning obstacles in discount (31.06% reduction) and tax (31.43% reduction) concepts, which showed the smallest improvements, may be attributed to the compounding difficulty of percentage operations within real-world contexts, a challenge also documented by Astutik (2015) and Nuraeni et al. (2020). These findings also support Brousseau's (2002) theory of didactical situations, which emphasizes that epistemological obstacles—deeply rooted in students' prior learning experiences—require prolonged and systematic instructional scaffolding to be fully overcome. The fact that profit/loss and percentage profit/loss concepts, which were addressed in the earliest instructional meeting, showed substantial improvements (45.83% and 50.00% reductions, respectively) suggests that the progressive sequencing of the learning trajectory, from fundamental economic exchange concepts to more complex financial applications, successfully established prerequisite knowledge before introducing advanced topics. This sequential alignment is consistent with Ausubel's (1968) theory of meaningful learning, which posits that new knowledge is most effectively acquired when it is explicitly connected to well-established prior knowledge structures. Furthermore, the overall reduction in learning obstacles across all topics, despite persistent challenges, confirms that ZPD-based scaffolding strategies—including peer tutoring, dialogic scaffolding, and gradual release of responsibility—constitute a pedagogically sound approach for addressing epistemological obstacles in mathematics instruction, particularly in contexts where students have historically struggled with contextual problem-solving and percentage operations (Wood, Bruner, & Ross, 1976; Hammond & Gibbons, 2005). Future iterations of the design should consider extending instructional time on discount and tax concepts, incorporating additional drills on percentage operations, and providing more intensive, individualized scaffolding for students who continue to exhibit procedural errors in multi-step word problems.

Improvement in Problem-Solving Skills and Persistent Challenges

The implementation data revealed a clear trajectory of progressive improvement in students' problem-solving skills across the four instructional meetings, with the percentage of students demonstrating adequate prerequisite knowledge increasing from approximately 70% in the first meeting to over 90% in the fourth meeting, correctly answering simple interest exercises. This progressive improvement supports Vygotsky's (1978) assertion that learning first occurs at the social level—through peer tutoring, teacher scaffolding, and collaborative group discussions—and then, as students internalize concepts and strategies, independently. The finding that the majority of students successfully calculated final prices after discounts and taxes by the second and third meetings indicates that the ZPD-based scaffolding strategies, including the use of authentic artifacts (marketplace screenshots, supermarket receipts) and peer-assisted learning, effectively moved most students from their potential development level to actual development level (Wood, Bruner, & Ross, 1976). However, the small minority of students who continued to struggle—approximately 15% still confused discount amounts with final prices and 10% continued to struggle with percentage multiplication—suggests that the ZPD is not uniform across all learners and that some students require more prolonged or intensive scaffolding to achieve the same developmental milestones (Hammond & Gibbons, 2005). This heterogeneity in learning trajectories is consistent with the differentiated instruction literature, which emphasizes that students enter the classroom with varied prior knowledge, cognitive processing capabilities, and motivational orientations, all of which influence the rate and depth of conceptual change (Tomlinson, 2014). The progressive nature of improvement observed across meetings also aligns with Hattie's (2009) synthesis of over 800 meta-analyses, which found that visible learning is characterized by incremental gains rather than sudden breakthroughs, and that even highly effective interventions typically yield moderate effect sizes when implemented over limited time frames.

Despite the overall improvement, four categories of learning obstacles persisted at the conclusion of the intervention, with discount and bruto-netto-tara concepts exhibiting the highest post-implementation LO percentages (41.67% for both). First, difficulties with word problem comprehension—where students misinterpreted questions or rushed without careful reading—confirm findings from Ana and Nusantara (2021) and Dila and Zanthy (2020) that linguistic and representational challenges often constitute primary barriers in social arithmetic, independent of computational proficiency. Second, persistent difficulties with percentage operations, even after targeted scaffolding in Meetings 2 and 3, suggest that percentage multiplication represents a "sticky" procedural knowledge that requires distributed practice across multiple contexts rather than massed practice within single instructional episodes (Sweller, 1988; Rohrer & Taylor, 2006). Third, procedural errors in multi-step problems indicate that students' working memory may become overloaded when required to simultaneously hold intermediate results, select appropriate operations, and monitor for calculation errors, a phenomenon consistent with cognitive load theory (Sweller, 1988). Fourth, a small number of students continued to face challenges with basic arithmetic operations (addition, subtraction, multiplication, division), revealing that some learning obstacles originate not in social arithmetic per se but in insufficient mastery of prerequisite mathematical skills—a finding that underscores the importance of diagnostic assessment and remedial scaffolding before introducing applied topics (Ausubel, 1968; Kilpatrick, Swafford, & Findell, 2001). The particularly high persistence of learning obstacles in discount and bruto-netto-tara concepts (41.67% for both) can be explained by the compound cognitive demands of these topics: students must interpret contextual information (e.g., "2% tara per karung"), select correct formulas (e.g., netto = bruto - tara; final price = original price - discount amount), perform percentage operations, and execute multi-step calculations—all while maintaining situational awareness of what the question is actually asking (Schoenfeld, 1985). To address these persistent challenges, future iterations of the didactical design should consider: (a) incorporating explicit instruction in word-problem reading comprehension strategies, such as paraphrasing, visualization, and question-identification (Fuchs et al., 2003); (b) implementing spaced practice schedules for percentage operations, integrated across all four meetings rather than isolated in early sessions; (c) reducing cognitive load by breaking multi-step problems into smaller, scaffolded sub-tasks with intermediate feedback; and (d) providing differentiated scaffolding protocols, including one-on-one tutoring or modified problem sets, for students who exhibit persistent procedural errors after whole-class instruction.

Improvement in Social Interaction and Design Revision

The substantial improvement in social interaction observed throughout the implementation provides empirical support for Vygotsky's (1978) sociocultural theory, which posits that cognitive development is fundamentally mediated by social interaction and collaborative dialogue. Students reported feeling more confident when peers used familiar language to explain mathematical concepts, confirming that the "more knowledgeable other" (MKO) need not be a teacher or adult but can effectively be a peer who has achieved the actual development level within the same age group (Wood, Bruner, & Ross, 1976). This finding aligns with Slavin's (1996) research on cooperative learning, which demonstrated that peer-assisted learning strategies not only improve academic achievement but also enhance students' self-efficacy, motivation, and willingness to participate in classroom discourse. The teacher's strategy of providing intensive scaffolding to group leaders before discussions—and the subsequent reduction in students' reluctance to present results and increase in active participation—operationalizes Tharp and Gallimore's (1988) four-stage model of ZPD development, where assistance from more capable others (stage 1) gradually transitions to self-assistance (stage 2), internalization (stage 3), and ultimately de-automatization (stage 4), characterized by autonomous performance without external support. Observations that by the third and fourth meetings, fewer groups exhibited over-reliance on specific individuals, and students were more willing to present their group findings without coercion, indicate successful progression through these stages. This progressive independence supports Hammond and Gibbons' (2005) assertion that effective scaffolding is not merely about providing support but about systematically withdrawing support as learners develop competence, thereby fostering self-regulated learning rather than dependency. The improved social interaction also addresses a key concern raised by teacher interviews during the diagnostic phase: teachers reported that students tended to be passive during classroom instruction and reluctant to ask questions, suggesting that the ZPD-based design effectively transformed the classroom social climate from teacher-centered to learner-centered.

Based on the retrospective analysis of persistent learning obstacles and residual social interaction challenges, the didactical design was systematically revised across five dimensions, reflecting the iterative nature of Didactical Design Research (DDR) as articulated by Suryadi (2013). First, deepening prerequisite material through more intensive review sessions addresses the diagnostic finding that many procedural errors originated in insufficient mastery of basic arithmetic operations, fractions, and percentage concepts—a foundational requirement for meaningful learning as emphasized by Ausubel (1968). Second, enriching student worksheets with more diverse and contextually relevant problem examples addresses the persistent difficulty students exhibit with word problem comprehension and knowledge transfer, ensuring that students encounter varied problem representations that facilitate the development of flexible rather than rigid knowledge structures (Kilpatrick, Swafford, & Findell, 2001). Third, providing additional illustrations and real-life stimuli (e.g., images comparing bruto, netto, and tara; examples of different tax types; comparisons of simple interest for loans versus savings) directly targets the epistemological obstacles identified by Brousseau (2002), where students' knowledge remains confined to limited contexts and cannot be transferred to new situations. These concrete visual aids serve as cognitive bridges, helping students form mental models of abstract relationships before manipulating symbolic representations. Fourth, improving group discussion management—by paying closer attention to duration and ensuring active participation from all group members—addresses the observation that some groups exhibited over-reliance on specific individuals, a phenomenon that Slavin (1996) identified as "free rider" or "dominant participant" effects that can undermine the benefits of cooperative learning when not actively managed. Fifth, increasing student motivation, particularly for less active learners, through dialogic scaffolding and positive reinforcement, aligns with self-determination theory (Deci & Ryan, 2000), which posits that autonomy, competence, and relatedness are essential psychological needs for intrinsic motivation. The application of these revisions iteratively to all four meetings, rather than as post-hoc additions, ensures that the empirically refined didactical design is coherent and theoretically grounded. The resulting design, documented in Table 2, demonstrates that DDR is not merely a design-and-implement model but a recursive process where each implementation cycle generates empirical evidence that informs subsequent refinement (Suryadi, 2013; Fauzi & Suryadi, 2020), ultimately producing a robust didactical framework for future implementation in similar educational contexts.

D. Conclusion

This study aimed to identify specific learning obstacles in social arithmetic among Indonesian junior high students, develop a Zone of Proximal Development (ZPD)-based didactical design tailored to those obstacles and students' learning trajectories, and evaluate the design's effectiveness in improving conceptual understanding, problem-solving skills, and social interaction. The most important findings revealed that eighth-grade students experienced high percentages of epistemological obstacles across all social arithmetic concepts, with the highest in gross, net, and tare (87.88%), simple interest (75.75%), and discount calculations (72.73%), and that students' mathematical understanding remained predominantly procedural rather than conceptual. Based on these findings, a systematically designed ZPD-based didactical design was developed, integrating Ausubel's meaningful learning theory, constructivist theory, and Vygotsky's ZPD framework, operationalized through four sequential instructional meetings, heterogeneous peer-led grouping, digital self-paced worksheets, a three-stage instructional structure, and Didactical-Pedagogical Anticipation (DPA) strategies. The evaluation demonstrated substantial effectiveness, with a normalized gain of 0.58 (moderate category), most pronounced reductions in percentage profit/loss (50.00%) and simple interest (46.58%), progressive improvement in problem-solving skills from 70% to over 90% mastery, and enhanced social interaction characterized by increased student confidence, reduced reluctance to present findings, and successful progression through Tharp and Gallimore's four-stage ZPD model.

The study contributes theoretically by extending the ZPD framework to social arithmetic instruction and demonstrating synergistic application of three learning theories; methodologically by validating DDR as a recursive, evidence-driven design approach; and practically by providing Indonesian mathematics teachers with a concrete, replicable instructional framework and curriculum developers with evidence-based recommendations for balancing contextual problem-solving with procedural mastery. However, the study has several limitations: a short intervention duration (4 meetings), a limited sample size at a single school, persistent learning obstacles with discount, gross-netto-tara concepts (41.67% post-implementation), difficulties in word problem comprehension, percentage operation errors, and multi-step procedural mistakes. Future research should implement the refined design over longer periods with larger, more diverse samples; incorporate explicit word-problem comprehension strategies, spaced practice for percentage operations, cognitive load reduction through task decomposition, and differentiated scaffolding protocols for struggling students; conduct component analysis and fidelity checks; adapt the framework to other mathematics topics; quantitatively measure changes in self-efficacy and mathematics anxiety using validated instruments; and conduct longitudinal studies to examine long-term retention and transfer of conceptual understanding to novel problem contexts.

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